We investigate a new class of Bayesian nonparametric priors derived by convolution mixture (composition of the product) $(\psi)$ r.v. by an independent $(\xi)$ r.v. belonging to the family of Generalized Gamma convolutions (B massacde, 1992). We rely on the study proposed in James (2006) and on recent results for posterior analysis of BNP priors obtained by normalization (James, 2005, 2008). We derive posterior analysis, predictive distribution and the specific form of the induced exchangeable partition probability function.

**Introduction**

Some famous BNP priors have been obtained in the last few years exploring families of infinitely divisible r.v.s produced by two well-known techniques to derive ID r.v.s by a given one. In Lijoi et al. (2005a) the Generalized Dirichlet process prior is obtained by exploiting the exponential family generated by the 1/2-Stable r.v. In Cerquetti (2007) and Lijoi et al. (2007) the normalized Generalized Gamma priors are shown to be the exponential family generated by the positive stable density for $\alpha > 0$.

Actually, beside finite convolutions and exponential tilting one can obtain ID distributions even by scale mixtures, and convolution mixtures. While in general a scale mixture of ID r.v.s generates a new density, characterized by a Lévy measure of the form $\psi(x) = \alpha \int_{0}^{\infty} x^{\lambda-1} \psi(\theta){\psi}_{\theta}(dx)$.

**Normalized Random Measures**

Let $S_0 \sim S_{\psi}(\theta)$ be the non-negative non-decreasing function called the Thorin measure, which uniquely determines the law of the $(\psi)$. It must satisfy conditions for the corresponding Laplace exponent to be finite, and it can be shown it is linked to the density of $\psi_0$ by the relationship $\psi(x) = \int_{0}^{\infty} e^{-xt} \psi(dx)$.

Notice that the class we are considering has been already deeply investigated in James (2006, see Section 5). Here we adhere to his notation but maintain Bondesson’s notation for the Thorin measure. Our results relies on posterior BNP analysis of GCC proposed in James et al. (2005, 2009).

For what follows suffice to deal with the general construction of homogenous normalized random measures. Let $(\psi_0)$ be a Poisson random measure on $(\mathcal{X}, \Sigma)$ with an homogeneous intensity measure $\psi_0(dx) = \psi_0(dx)$ where $\psi_0$ is a fixed non-atom non-negative probability measure. Given a positive measurable function $\phi \in L_1(\mathcal{X})$ a complete random measure (CRM) on $(\mathcal{X}, \Sigma)$ is obtained as $\psi(x) = \int_{0}^{\infty} e^{-xt} \psi(dx)$.

**Posterior Analysis of Normalized Generalized Gamma (GGC) Priors**

By James et al. (2005, 2009) we know that to the posterior analysis of normalized BNP priors is the positive r.v. $u_{\alpha} = \Gamma(\alpha)/T$ for $\alpha > 0$, $\alpha$ in $(0,1)$. Its density for $T > GGC(0,\alpha,\theta) = 0$ is given by $f_{\psi}(\alpha) = \frac{\Gamma(\alpha)}{T^{\alpha} \Gamma(\alpha)} f(\alpha)$.

Now let $f_{\psi}(\alpha) = \mu(\alpha)$ for $\psi = GGC(\alpha,\theta)$.

By construction $f_{\psi}(\alpha)$ admits the following representation:

$$f_{\psi}(\alpha) = \mu(\alpha) + \int_{0}^{\infty} 1 \frac{\Gamma(\alpha)}{\Gamma(\alpha+1)} \mu(\alpha+1) (\alpha+1)^{1/\alpha}$$

**SELECTED REFERENCES**


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**On a class of Bayesian nonparametric priors derived by subordination of Stable processes**

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